Exercises for Further Graphics (Lectures 1-4)

All work to be submitted by email in a single PDF, no less than 48 hours before supervision.

1. **Terms and Concepts**

   Voronoi Diagrams

   a. What is *equiangularity*?
   b. What is the *empty circle property*?
   c. Describe how to use hardware acceleration to swiftly compute Voronoi diagrams. What are the limitations of this approach?

   Topology

   a. Define the *Euler characteristic*
   b. Define the term *angle deficit*
   c. State the *Poincaré Theorem*, which links the geometry of a surface to its topology
   d. State Descartes' *Theorem of Total Angle Deficit*, which links angle deficit across a surface to its Euler characteristic

   Curvature

   a. The *one-ring* of a vertex is the (usually ordered) set of vertices which lie exactly one edge away from a given vertex on a polyhedral surface. Given a vertex V with one-ring $\{v_0,\ldots,v_{n-1}\}$, give a formula for the discrete curvature of the surface at V.

2. **Signed Distance Functions**
   Give signed distance functions for:
   a. A cone
   b. An igloo
   c. An arbitrary tetrahedron
   d. The spiral of a corkscrew

3. **Particle systems**
   a. Explain why the forward difference method (Euler's Method) is error-prone, and describe 1-3 strategies used in realtime graphics today to mitigate error.

4. **Bezier curves**
    a. Why is a Bezier curve contained entirely within the convex hull of its control points?
    b. Give real-world examples of C0, C1, C2 continuity
    c. Prove that the linear interpolation of two linear interpolations is, in fact, a Bezier quadratic.

5. **B-Splines**
    a. Show that the B-spline with k = 3 and knot vector `[ 0 0 0 1 1 1 ]` is equivalent to the quadratic Bezier curve.
    b. Give a knot vector and value of *k* which would describe a uniform B-spline equivalent to a cubic Bezier curve.
    c. Derive the formula of and sketch a graph of $N_{3,3}(t)$, the third of the quadratic B-spline basis functions, for the knot vector `[ 0 0 0 1 3 3 4 5 5 5 ]`.